

Chapter 14 – Radians

14.1 Basic Relationships

When working with trigonometrical functions we often find the need to convert an angle in degrees to radians and vice versa.

Recall: $1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57.3^\circ$

(or $\pi \text{ radians} = 180 \text{ degrees}$)

To convert x° to radians we have:

$$x^\circ = x \left(\frac{\pi}{180} \right) \text{ radians}$$

and to convert x radians to degrees we have:

$$x \text{ radians} = \left[x \left(\frac{180}{\pi} \right) \right]^\circ$$

14.2 Converting Using C and D scales

As degrees are converted to radians by multiplying the angle in degrees by $\frac{\pi}{180} = 0.01745$, many Slide Rule have a mark labeled δ (or with some other symbol) at '1745' on the C and D scales (also CF and DF scales). Thus, if we set the index of the C scale above the δ on the D scale we have its radian equivalent on the D scale.

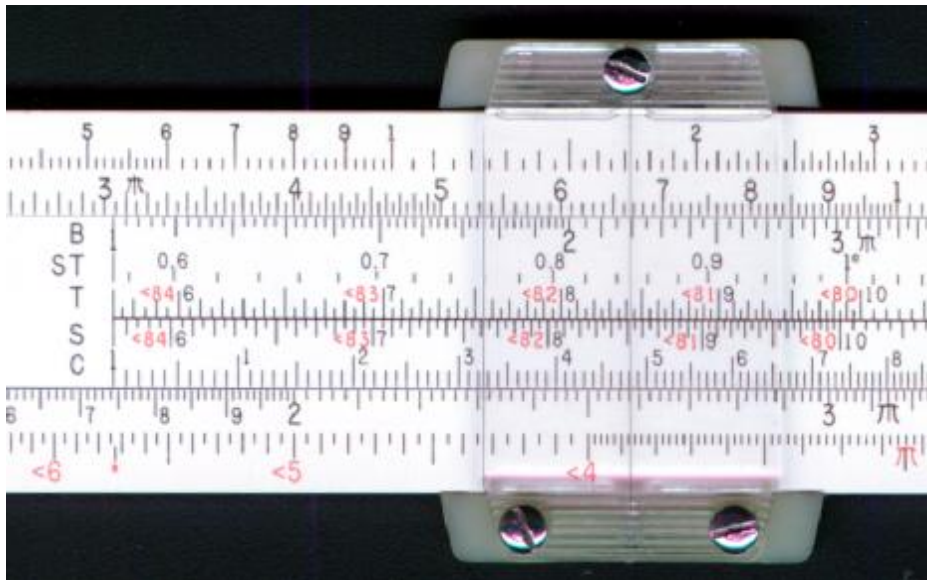


Fig 14-1

Example: $14.8^\circ = 0.258 \text{ radians}$ (Fig. 14-1)

1. Set the hair line over δ on the D scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 14.8 on the C scale.
4. Under the hair line read off 0.258 on the D scale as the answer.

Note:

- (a) The above setting of the Slide Rule would also give us $1.48^\circ = 0.0258 \text{ radians}$, $148^\circ = 2.58 \text{ radians}$, etc. This is because the relationship between degrees and radians is a simple linear one. (e.g. $57.3^\circ = 1 \text{ radian}$ $\therefore 114.6^\circ = 2 \text{ radians}$, etc.)

- (b) For the C and D scales set as above (from steps 1 and 2), if the angle in degrees on the C scale takes us off the end of the D scale, we find the angle in degrees on the CF scale and hence its value in radians on the DF scale. For example, observe on Fig. 14-1 under the hair line, 46.5° on the CD scale gives 0.81 radians on the DF scale.
- (c) To convert from radians to degrees, we find the radian value on the D or DF scale and its degree equivalent is thus read off the C or CF scale.

Exercise 14(a)

Convert to Radians:

- | | | | |
|-------|------------------|------|----------------|
| (i) | $24^\circ =$ | (iv) | $125^\circ =$ |
| (ii) | $53^\circ 30' =$ | (v) | $1.25^\circ =$ |
| (iii) | $69^\circ 24' =$ | (vi) | $250^\circ =$ |

Convert to Degrees:

- | | | | |
|--------|---------------|-------|-----------------|
| (vii) | 1 radian = | (x) | 0.065 radians = |
| (viii) | 0.8 radians = | (xi) | 6.5 radians = |
| (ix) | 3.8 radians = | (xii) | 1.3 radians = |

14.3 Converting using the ST Scale

For small angles (i.e. below about 5° or 6°), the sine, tangent and radian value of an angle are all same to at least three figures. Thus, for an angle in degrees on the ST scale, its radians equivalent is read directly off the D scale. The actual graduations on the ST scale are only from 0.574° to 5.74° , 57.4° to 574° , 0.0574 to 0.574° , etc. This is because we have a linear relationship between degrees and radians, which is of course not so for an angle in degrees and its sine, cosine, tangent, etc.

Example $75^\circ = 1.31$ radians (Fig 14-2)

1. Set the hair line over 75 (marked as 0.75) on the ST scale.
2. Under the hair line read off 1.31 on the D scale as the answer.

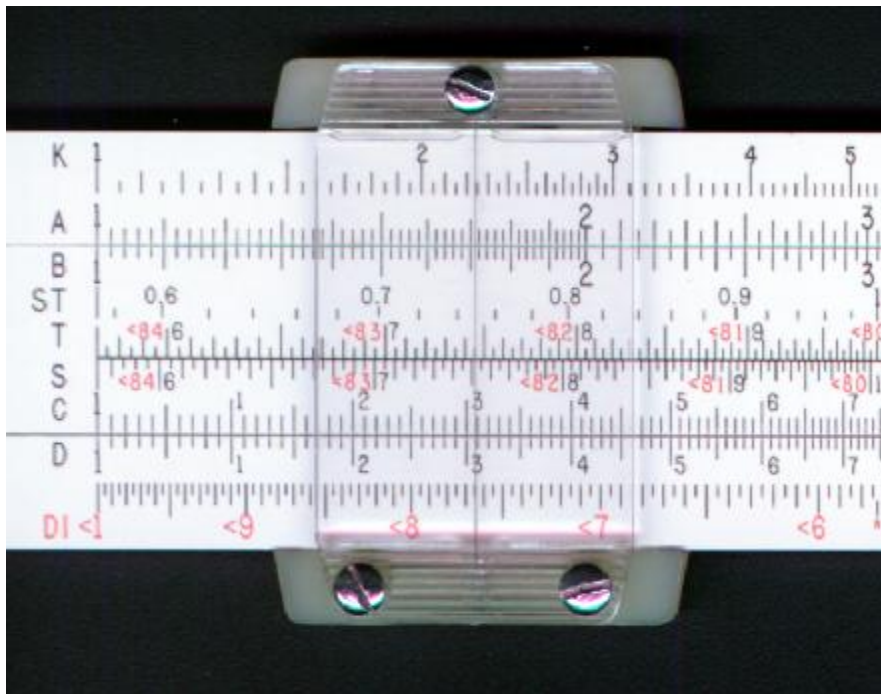


Fig 14-2

Note:

- (a) In the above example we could take the same setting on the ST scale as 7.5° , 750° , 0.75° , etc., in which case the equivalent value in radians would have been 0.131, 13.1, 0.0131, etc. respectively.

- (b) To convert from radians to degrees, we find the radians value on the D scale and its degrees equivalent is read off the ST scale.

Exercise 14(b)

Covert to radians:

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|-------|---------------|------|-----------------|
| (i) | $4.5^\circ =$ | (iv) | $0.36^\circ =$ |
| (ii) | $36^\circ =$ | (v) | $91.36^\circ =$ |
| (iii) | $360^\circ =$ | (vi) | $12' =$ |

Convert to degrees:

- | | | | |
|--------|------------------|-------|--------------------|
| (vii) | 0.65 radians = | (x) | 5 radians = |
| (viii) | 1.2 radians = | (xi) | 0.625 radians = |
| (ix) | 0.12 radians = | (xii) | 0.0314 radians = |

14.4 Arc Length and Area of a sector

Recall the formulae:

Arc Length, $L = r\theta$ (for $r =$ radius and θ angle in radians)

Area of a Sector, $A = \frac{1}{2}r^2\theta$

Thus, if the angle θ was given in degrees (instead of radians), we could still evaluate the above quite readily.

Example: $L = r\theta$ for $r = 14$ and $\theta = 60^\circ$ (Fig. 14-3)

- Set the hair line over 60° (marked as 0.6) on the ST scale.
- Place the left index of the C scale under the hair line.
- Reset the hair line over 14 on the C scale.
- Under the hair line read off 14.65 on the D scale as the answer.
 $\therefore L = 14.65$

Note: The area of a sector could be similarly found but would necessitate a second movement of the slide to multiply by the extra factor of r and the $\frac{1}{2}$. A better method of obtaining the area of a sector is found in Unit 20.

Exercise 14(c)

Find the arc length given:

- | | | | |
|------|-------------------------------------|-------|---------------------------------------|
| (i) | $r = 2.5$ cm, $\theta = 45^\circ$. | (iii) | $r = 15$ cm, $\theta = 2^\circ 30'$. |
| (ii) | $r = 4$ cm, $\theta = 120^\circ$. | (iv) | $r = 6.8$ cm, $\theta = 315^\circ$. |

Find the area of the sector given:

- | | | | |
|-----|-------------------------------------|------|------------------------------------|
| (v) | $r = 2.5$ cm, $\theta = 45^\circ$. | (vi) | $r = 4$ cm, $\theta = 120^\circ$. |
|-----|-------------------------------------|------|------------------------------------|